

10MAT31
Third Semester B.E. Degree Examination, June/July 2017 Engineering Mathematics - III

## Time: 3 hrs.

Max. Marks: 100
Note: Answer any FIVE full questions, selecting atleast TWO questions from each part.

## PART - A

1 a. Obtain Fourier series for the function $f(x)$ given by
$f(x)=\left\{\begin{array}{ll}1+\frac{2 x}{\pi}, & -\pi \leq x \leq 0 \\ 1-\frac{2 x}{\pi}, & 0 \leq x \leq \pi\end{array}\right.$.
Hence deduce that $\frac{1}{1^{2}}+\frac{1}{3^{2}}+\frac{1}{5^{2}}+\ldots . .=\frac{\pi^{2}}{8}$.
(06 Marks)
b. Obtain Fourier half range Cosine series for the function $f(x)=x \sin x$ in $(0, \pi)$. Hence show that $\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-\ldots . \infty=\frac{\pi-2}{4}$.
(07 Marks)
c. Obtain the constant term and the co-efficient of the first sine and cosine terms in the Fourier series of $f(x)$ as given in the following table.
(07 Marks)

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(\mathrm{x})$ | 9 | 18 | 24 | 28 | 26 | 20 |

2 a. Find the Fourier transform of $\mathrm{e}^{-\mathrm{a}^{2} x^{2}}, \mathrm{a}<0$. Hence deduce that $\mathrm{e}^{-\mathrm{x}^{2} / 2}$ is self reciprocal in respect of Fourier transform.
(06 Marks)
b. Find the Fourier sine transform of $\mathrm{e}^{-\mid x}$. Hence show that

$$
\int_{0}^{\infty} \frac{x \sin m x}{1+x^{2}} d x=\frac{\pi e^{-m}}{2}, m>0
$$

(07 Marks)
c. Find the Fourier Cosine transform of $\mathrm{f}(\mathrm{x})=\frac{1}{1+\mathrm{x}^{2}}$.
(07 Marks)

3 a. Obtain various possible solutions of the one dimensional Heat equation $\frac{\partial u}{\partial t}=c^{2} \frac{\partial^{2} u}{\partial x^{2}}$ by the method of separation of variables.
(06 Marks)
b. Obtain the D'Alembert's solution of the wave equation $\frac{\partial^{2} u}{\partial t^{2}}=c^{2} \frac{\partial^{2} u}{\partial x^{2}}$. Subject to the conditions $u(x, 0)=f(x)$ and $\frac{\partial u}{\partial t}(x, 0)=0$.
(07 Marks)
c. Obtain various possible solutions of the two dimensional Laplace equation $u_{x x}+u_{y y}=0$ by the method of separation of variables.
(07 Marks)
4 a. Fit a parabola $y=a x^{2}+b x+c$ to the following data :
(06 Marks)

| x | 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| y | 1 | 3 | 7 | 13 | 21 | 31 |

b. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs 5,760 to invest and has space for at most 20 items. A fan and sewing machine sost 360 and Rs 240 respectively. He can sell a fan at a profit of Rs 22 and sewing machine at a profit of Rs 18. Assuming that he can sell whatever he buys, how should he invest his money in order to maximize his profit? Translate the problem into LPP and solve it graphically.
(07 Marks)
c. Use Simplex method to solve the following LPP

Minimize $Z=x_{1}-3 x_{2}+3 x_{3}$
Subject to $3 x_{1}-x_{2}+2 x_{3} \leq 7$

$$
2 x_{1}+4 x_{2} \geq-12
$$

$$
-4 x_{1}+3 x_{2}+8 x_{3} \leq 10
$$

$$
\mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3} \geq 0
$$

(07 Marks)

## PART - B

a. Using Newton - Raphson method, find the value of $\sqrt[3]{18}$ correct to 2 decimals, assuming 2.5 as the initial approximation.
(06 Marks)
b. Apply Gauss - Seidal iteration method to solve the following equations :
$3 x+20 y-z=-18 ; \quad 2 x-3 y+20 z=25 ; \quad 20 x+y-2 z=17$.
(07 Marks)
c. Find the largest Eigen - value and the corresponding Eigen - vector for the matrix $\left[\begin{array}{ccc}1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10\end{array}\right]$ with initial approximation $\left[\begin{array}{ll}1 & 1\end{array} 0\right]^{\mathrm{T}}$.
(07 Marks)

6 a. Determine $f(x)$ as a polynomial in $x$ for the following data by using Newton's divided difference formula.
(06 Marks)

| $x$ | -4 | -1 | 0 | 2 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 1245 | 33 | 5 | 9 | 1335 |

b. From the data given in the following table, find the number of students who obtained
i) less than 45 marks and ii) between 40 and 45 marks.
(07 Marks)

| Marks | $30-40$ | $40-50$ | $50-60$ | $60-70$ | $70-80$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| No. of students | 31 | 426 | 51 | 35 | 31 |

c. Evaluate $\int_{4}^{5.2} \log _{\mathrm{e}} \mathrm{x} d \mathrm{dx}$ by Weddle's rule.
(07 Marks)
a. Solve the Laplace equation $u_{\mathrm{xx}}+\mathrm{u}_{\mathrm{yy}}=0$, given that the boundary values for the following square mash.
(06 Marks)

b. Evaluate the pivotal values of the equation $u_{t t}=16 u_{x x}$, taking $h=1$ upto $t=1.25$. The boundary conditions are $\mathrm{u}(0, \mathrm{t})=\mathrm{u}(5, \mathrm{t})=0, \mathrm{u}_{\mathrm{i}}(\mathrm{x}, 0)=0$ and $\mathrm{u}(\mathrm{x}, 0)=\mathrm{x}^{2}(5-\mathrm{x})$.
(07 Marks)
c. Solve $\frac{\partial u}{\partial t}=\frac{\partial^{2} u}{\partial x^{2}}$ in $0<x<5, t \geq 0$, given that $u(x, 0)=20, u(0, t)=0, u(5, t)=100$. Compute u for the time - step with $\mathrm{h}=1$ by ${ }^{\circ} \mathrm{Crank}-$ Nicholson method.
(07 Marks)
8 a. Find the Z - transform of the following :
i) $(\mathrm{n}+1)^{2}$
ii) $\sin (3 n+5)$
iii) $\mathrm{n}_{\mathrm{c}_{\mathrm{p}}}(0 \leq \mathrm{p} \leq \mathrm{n})$.
(06 Marks)
b. If $u(z)=\frac{2 z^{2}+3 z+12}{(z-1)^{4}}$. Find $u_{0}, u_{1}, u_{2}, u_{3}$.
c. Solve $y_{n+2}+4 y_{n+1}+3 y_{n}=3^{n}$ with $y_{0}=0, y_{1}=1$, using $Z$ - transforms.
(07 Marks)


Third Semester B.E. Degree Examination, June/July 2017

## Electronic Circuits

Time: 3 hrs .
Max. Marks: 100

## Note: Answer FIVE full questions, selecting at least TWO questions from each part.

## PART - A

1 a. What is UJT? With the help of relevant diagram, explain the construction and operational principle of a UJT.
(08 Marks)
b. For the fixed biased circuit of Fig.Q1(b), determine the operating point (given that $\beta=100$, $\mathrm{V}_{\mathrm{BE}}=0.7 \mathrm{~V}$ ). Also draw the load line for the circuit.


Fig.Q1(b)
(07 Marks)
c. Explain thermal runaway as referred to transistor.
(05 Marks)
2 a. With the help of neat diagrams, explain the construction and characteristics of N -channel depletion MOSFET.
(10 Marks)
b. Fig.Q2(b) shows a biasing configuration using DE-MOSFET, given that the saturation drain current is 8 mA and the pinch off voltage is -2 V . Determine the value of the gate source voltage, drain current and drain source voltage.


Fig.Q2(b)
(05 Marks)
c. Explain the operation of CMOS inverter.

3 a. Define the following terms:
i) Responsivity
ii) Noise equivalent power (NEP)
iii) Detectivity and Dee star
iv) Quantum efficiency
v) Response time
(05 Marks)
b. What is a photo transistor? Draw the schematic symbol of a photo transistor. Explain its V-I characteristics.
(05 Marks)
c. A photodiode has a noise current of 1 fA , responsivity figure of $0.5 \mathrm{~A} / \mathrm{W}$, active area of $1 \mathrm{~mm}^{2}$ and rise time of 3.5 ns . Determine its:
i) NEP
ii) Detectivity
iii) $\mathrm{D}^{*}$
iv) Quantum efficiency at 850 nm .
(05 Marks)
d. What are opto couplers? Explain the important characteristic parameters of opto couplers.
(05 Marks)
4 a. Draw the generalized h-parameter model of a transistor based amplifier and derive the expression for:
i) Current gain
ii) Input impedance
iii) Voltage gain
iv) Output admittance
(10 Marks)
b. With neat figure, explain the operation of Darlington Amplifier.
(05 Marks)
c. What are cascade amplifiers? What are the advantages of cascade amplifiers?
(05 Marks)

## PART - B

5 a. Explain classification of large signal amplifiers as class $A$, class $B$, class $C$ and class $A B$ amplifiers.
(04 Marks)
b. What are the advantages of negative feedback?
(04 Marks)
c. Derive the relevant expressions to prove that input resistance increases and output resistance reduces in case of a voltage series feedback.
(08 Marks)
d. The total harmonic distortion of an amplifier reduces from $10 \%$ to $1 \%$ on introduction of $10 \%$ negative feedback. Determine the open loop and closed loop gain values.
(04 Marks)
6 a. Explain the Barkhausen criterion as referred to oscillators.
(05 Marks)
b. With a neat diagram, explain the operation of voltage controlled Hartley oscillator.(07 Marks)
c. With a neat circuit and relevant waveforms, explain the operation of monostable multivibrator using IC 555 timer.
(08 Marks)
7 a. Name the constituent parts of a basic linearly regulated power supply. Briefly describe the function of each of the constituent parts.
(03 Marks)
b. Define: i) Load regulation; ii) Line regulation, iii) Ripple rejection factor with reference to regulated power supplies.
(04 Marks)
c. With neat figure, explain the working of a Buck Regulator.
(08 Marks)
d. Refer to the three terminal regulator circuit of Fig.Q7(d). Determine: (i) Load current, (ii) Current through LM7812, (iii) Current through external transistor, (iv) Power dissipated in LM7812. Take $\mathrm{V}_{\mathrm{BE}\left(\mathrm{Q}_{1}\right)}=0.7 \mathrm{~V}$.


Fig.Q7(d)
(05 Marks)
8 a. Define the following: i) CMRR, ii) PSRR, iii) Slew rate, iv) Band width, v) Open loop gain of an op-amp.
(05 Marks)
b. With a neat figure, explain the operation of a peak detector.
(07 Marks)
c. With a neat figure and relevant waveforms, explain the working of relaxation oscillator circuit using op-amp.
(08 Marks)


Third Semester B.E. Degree Examination, June/July 2017 Discrete Mathematical Structures

Time: 3 hrs .

Max. Marks:100

## Note: Answer FIVE full questions, selecting at least TWO questions from each part.

## PART-A

1 a. Using the Venn diagram, prove that $\mathrm{A} \Delta(\mathrm{B} \Delta \mathrm{C})=(\mathrm{A} \Delta \mathrm{B}) \Delta \mathrm{C}$
(06 Marks)
b. In a survey of 60 people it was found that 25 read weekly magazines, 26 read forthrightly magazines, 26 read monthly magazines, 9 read both weekly and monthly magazines, 11 read both weekly and fortnightly magazines, 8 read fortnightly and monthly magazines and 3 read all three magazines, Find
i) The number of people who read at least one of the three magazines and
ii) The number of people who read exactly one magazine.
(07 Marks)
divisible by 3 is chosed and $B$ is the event that the chosen number exceeds 10 , determine the $P_{r}(A), P_{r}(B), P_{r}(A \cap B)$ and $P_{r}(A \cup B)$.
(07 Marks)
2 a. Prove the following logical equivalence without using truth tables $[(p \vee q) \vee(\neg p \wedge \neg q \wedge r)] \Leftrightarrow(p \vee q \vee r)$
(06 Marks)
b. Define tautology. Examine whether the compound proposition is a tautology. $[\mathrm{p} \vee(\mathrm{q} \wedge \mathrm{r})] \vee \neg[\mathrm{p} \vee(\mathrm{q} \wedge \mathrm{r})]$.
(07 Marks)
c. State the converse, inverse and contra positive of the conditional "If two lines are parallel then they are equidistant"
(07 Marks)
3 a. For the universe of all real numbers, define the following open statements, $p(x): x \geq 0, q(x): x^{2} \geq 0, r(x): x^{2}-3>0$.
Determine the truth value of the following statements.
i) $\exists x, p(x) \wedge q(x)$
ii) $\forall \mathrm{x}, \mathrm{p}(\mathrm{x}) \rightarrow \mathrm{q}(\mathrm{x})$
iii) $\forall x, q(x) \rightarrow r(x)$
(06 Marks)
b. Find whether the following argument is valid. If a triangle has two equal sides, then it is isosceles. If a triangle is isosceles, then it has two equal angles
the triangle $A B C$ does not have two equal sides
$\therefore \mathrm{ABC}$ does not have two equal sides
(07 Marks)
c. Give:
i) a direct proof
ii) an indirect proof and
iii) Proof by contradiction for the following statement. "If $m$ is an even integer, then $m$ +5 is an odd integer".
(07 Marks)

4 a. Prove the following result by mathematical induction
$1^{2}+2^{2}+3^{2}+\ldots \ldots+n^{2}=\frac{1}{6} n(n+1)(2 n+1)$.
(06 Marks)
b. Find an explicit definition of the sequence defined recursively by $a_{1}=7, a_{n}=2 a_{n-1}+1$ for $\mathrm{n} \geq 2$.
(07 Marks)
c. Let $F_{n}$ denote the $n^{\text {th }}$ Fibonacci number prove that $\sum_{i=1}^{n} \frac{F_{i-1}}{2^{i}}=1-\frac{F_{n+2}}{2^{n}}$.
(07 Marks)

## PART - B

5 a. Define Cartesian product of two sets, Let $A=\{a, b, c\}, B=\{1,2\}$ and $C=\{x, y, z\}$, Find $\mathrm{A} \times(\mathrm{B} \cup \mathrm{C})$ and $(\mathrm{A} \times \mathrm{B}) \cup \mathrm{C}$.
(06 Marks)
b. Let $A=\{1,2,3,4\}$ and $B=\{a, b, c\}$ Find
i) Number of relations from A to B
ii) Number of one - to - one relations from A and B
iii) Number of on to functions from $A$ to $B$.
(07 Marks)
c. Let $\mathrm{f}: \mathrm{R} \rightarrow \mathrm{R}$ and $\mathrm{g}: \mathrm{R} \rightarrow \mathrm{R}$ be defined by $\mathrm{f}(\mathrm{x})=3 \mathrm{x}+2, \mathrm{~g}(\mathrm{x})=\frac{1}{2}(\mathrm{x}-3)$. Find $\mathrm{f}^{1}, \mathrm{~g}^{-1}$ and $\mathrm{f}^{1} 0 \mathrm{~g}^{-1}$.
(07 Marks)
6 a. Let $\mathrm{A}=\{1,2,3,4,6\}$ and R be a relation on A defined by aRb if and only if "a is a multiple of $b$ ". Write down the relation matrix $M(R)$ and draw its diagraph.
(06 Marks)
b. Define equivalence relation. Let $S$ be the set of all non-zero integers and $A=S \times S$ on $A$, define the relation $R$ by $(a, b) R(c, d)$ if and only if $a d=b c$. Show that $R$ is an equivalence relation.
(07 Marks)
c. Let $A=\{1,2,3,4,6,8,12\}$. On $A$, define the partial orderly relation $R$ by $x R y$ if and only of "x divides $y$ ". Draw the Hasse diagram for R.
(07 Marks)

7 a. If * is an operation on 2, defined by $x^{*} y=x+y+1$. Prove that $\left(2,{ }^{*}\right)$ is an abelian group.
(06 Marks)
b. Define subgroup of a group. Prove that the intersection of two subgroups of a group is a sub group of the group.
(07 Marks)
c. For a group $G$, prove that the function $f: G \rightarrow G$ defined by $f(a)=a^{-1}$ is an isomorphism if and only if $G$ is abelian.
(07 Marks)

8 a. The encoding function $\mathrm{E}: \mathrm{Z}_{2}^{2} \rightarrow \mathrm{Z}_{2}^{5}$ is given by the generator matrix

$$
G=\left[\begin{array}{lllll}
1 & 0 & 1 & 1 & 0 \\
0 & 1 & 0 & 1 & 1
\end{array}\right]
$$

i) Determine all the code words.
ii) Find the associated parity check matrix $H$.
(06 Marks)
b. Prove that $(z, \oplus, \otimes)$ is a ring with binary operations. $x \oplus y=x+y+1, x \otimes y=x+y+x y$, $\forall x, y \in z$.
(07 Marks)
c. Show that $\mathrm{Z}_{6}$ is an integral domain.
(07 Marks)


Third Semester B.E. Degree Examination, June/July 2017 Data Structures with C

Time: 3 hrs.
Max. Marks: 100

## Note: Answer FIVE full questions, selecting at least TWO questions from each part.

## PART - A

1 a. List and define the criteria's that an algorithm must satisfy. Write an algorithm and its C code for selection sort.
(08 Marks)
b. Define dynamic memory allocation. What are the benefits of dynamic memory allocation? Explain the memory allocation functions with example.
(07 Marks)
c. Find the space complexity and time complexity for the following function. Assume 32-bit machine.
float rsum (float list [ ], int n)
\{
if ( n )
return rsum (list, $n-1)+\operatorname{list}[\mathrm{n}-1]$;
return 0 ;
\} (05 Marks)
2 a. Develop a structure to represent the planets in the solar system. Each planet has fields for the planet's name, its distance from sun, and the number of moons it has. Initialize items in each of the fields for the planets: Earth and Venus.
(04 Marks)
b. Write a C program to add two polynomials.
(10 Marks)
c. Give the ADT of sparse matrix. Write a function to transpose a sparsematrix.
(06 Marks)
3 a. Define queue. List and define the different types of queues. Write the implementation of primitive operations of linear queue.
b. Write a C program to evaluate a given postfix expression.
c. Convert the following infix expression into postfix and prefix expression:

$$
(\mathrm{a}+\mathrm{b}) * \mathrm{~d}+\mathrm{e} /(\mathrm{f}+\mathrm{a} * \mathrm{~d})+\mathrm{c}
$$

(04 Marks)
4 a. Write a C program to implement a stack using linked list.
(06 Marks)
b. Write a function for inverting a simply linked list and a function for finding the length of a circular linked list.
(06 Marks)
c. Give a node structure for sparse matrices. Write the linked representation for the following sparse matrix.

$$
\left[\begin{array}{llll}
2 & 0 & 0 & 0 \\
4 & 0 & 0 & 3 \\
0 & 0 & 0 & 0 \\
8 & 0 & 0 & 1 \\
0 & 0 & 6 & 0
\end{array}\right]
$$

## PART - B

5 a. List and explain the different types of representation of trees with an example. (06 Marks)
b. Write the C implementation of inorder, preorder and postorder traversals. Illustrate with an example.
(08 Marks)
c. Suppose that we have the following key values $7,16,49,82,5,31,6,2,44$. Write out the max heap and min heap after each value is inserted into the heap.

6 a. With an example, explain selection trees.
(06 Marks)
b. With an example explain weighting rule for union and collapsing rule for find operation.
(08 Marks)
c. Construct a binary search tree by using the following inorder and preorder traversals.

Inorder : BCAEDGHFI
Preorder : ABCDEFGHI
(06 Marks)
7 a. Briefly explain the height-biased leftiest trees and weight-biased leftiest trees with example.
(08 Marks)
b. What is binomial heap? Explain the steps involved in the deletion of min element from a binomial heap.
(08 Marks)
c. List and define the different types of pairing heaps. Explain meld operation of pairing heaps with an example.

8 a. What is an AVL tree? Write the algorithm to insert an item into AVL tree. Explain LR rotation with an example.
(10 Marks)
b. Write short notes on the following:
i) Red-black trees
ii) Splay trees

## USN



# Third Semester B.E. Degree Examination, June/July 2017 Object Oriented Programming with C++ 

## Time: 3 hrs .

Max. Marks: 100

## Note: 1. Answer FIVE full questions, selecting at least TWO questions from each part. 2. Programs must be neatly documented.

## PART - A

1 a. What is reference variable? Explain with an example and write a program to swap values of two variables using reference variable.
(05 Marks)
b. Describe function overloading and write a program using overloaded function area to find area of circle, triangle and rectangle.
(05 Marks)
c. What is an inline function? What is the advantage of having a function inline? Write a C++ program to find maximum of three integers using inline function maximum (). (05 Marks)
d. Illustrate with an examples, different data types supported by C++ language. ( 05 Marks)

2 a. What is data hiding? Write a C++ program to create a class complex, to add given two complex numbers and use following member functions, readData( ), dispData( ) and computeData().
(05 Marks)
b. What are constructor and destructor? Can you overload constructor and destructor? Justify.
(08 Marks)
c. What are static members of a class? Illustrate with an example and write a program to count the number of object created.
(07 Marks)
3 a. What is friend function? Explain. Write a C +4 program using Bridge friend function small( ) to find smallest of two numbers.
(06 Marks)
b. What is generic function and template instantiation? Write a C++ program using generic function swap( ) to exchange values of two integers, doubles and characters, and prints the values before and after swapping.
(07 Marks)
c. What is operator overloading? Why it is required? Write a C++ program to overload the operators '+' to add two complex numbers, '<<' to display complex numbers and ">>" to read complex numbers, using friend functions.
(07 Marks)
4 a. What is inheritance? Explain the differences between the access specifier flags / visibility modes.
(06 Marks)
b. Explain single inheritance and multiple inheritance with the suitable diagram and syntaxes.
(08 Marks)
c. Write a C++ program to create a class called CSE (Name and USN) and using inheritance crate derived classes, UG (fee, stipend) and PG (fee, stipend) from it.
(06 Marks)

## PART - B

5 a. Explain constructor and destructor functions and how to pass arguments to constructors along with multilevel inheritance.
(10 Marks)
b. What is virtual base class? Explain with the suitable diagram and program.
(10 Marks)
6 a. What is runtime polymorphism? How to achieve it? With the suitable example program explain the same.
(10 Marks)
b. Explain pure virtual function and abstract class with the suitable program.
(10 Marks)
7 a. Explain input output manipulator with the suitable example.
(10 Marks)
b. Explain file operations with examples.
(10 Marks)
8 a. What is an exception? Explain exception handling options with an example.
(10 Marks)
b. What is STL? What STL consists of? Explain in detail vector class.


MATDIP301

Third Semester B.E Degree Examination, June/July 2017 Advanced Mathematics - I

Time: 3 hrs.
Max. Marks: 100

## Note: Answer any FIVE full questions.

1 a. Express: $\frac{1}{(2+\mathrm{i})^{2}}-\frac{1}{(2-\mathrm{i})^{2}}$ in the form of $\mathrm{a}+\mathrm{i} \mathrm{b}$.
(07 Marks)
b. Find the modulus and amplitude of the complex number $1-\cos \alpha+i \sin \alpha$.
(06 Marks)
c. Express the complex number $\sqrt{3}+\mathrm{i}$ in the polar form.
(07 Marks)
2 a. Find the $\mathrm{n}^{\text {th }}$ derivative of $\log (\mathrm{ax}+\mathrm{b})$.
(07 Marks)
b. Find the $n^{\text {th }}$ derivative of $\frac{x}{(x-1)(2 x+3)}$.
(06 Marks)
c. If $y=\sin ^{-1} x$, prove that : $\left(1-x^{2}\right) y_{n+2}-(2 n+1) x y_{n+1}-n^{2} y_{n}=0$.
(07 Marks)
3 a. Using Taylor's theorem, expand $\sin x$ in power of $(x-\pi / 2)$.
(07 Marks)
b. Obtain the Maclaurin's series expansion of the function $\sqrt{1+\sin 2 \mathrm{x}}$ up to the term containing $\mathrm{x}^{4}$.
c. State and prove Euler's theorem.
(07 Marks)
4 a. Find the total derivative of $z=x y^{2}+x^{2} y$ where $x=a t, y=2 a$ at, and also verify the result by direct substitution.
(07 Marks)
b. If $u=f(y-z, z-x, x-y)$ prove that : $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$.
(06 Marks)
c. if $x=u(1-v)$ and $y=u v$, find $J=\frac{\partial(x, y)}{\partial(u, v)}$ and $J^{\prime}=\frac{\partial(u, v)}{\partial(x, y)}$ and also verify $J \cdot J^{\prime}=1$.
(07 Marks)
5 a. Obtain the reduction formula for $\int \cos ^{n} x \cdot d x$.
(07 Marks)
b. Evaluate : $\int_{0}^{2} \frac{x^{4}}{\sqrt{4-x^{2}}} \cdot d x$.
(06 Marks)
c. Evaluate : $\int_{1}^{2} \int_{1}^{3} x y^{2} d x d y$.
(07 Marks)

6 a. Evaluate : $\int_{0}^{1} \int_{0}^{\sqrt{1-x^{2}}} \int_{0}^{\sqrt{1-x^{2}-y^{2}}} x y z d z d y d x$.
(07 Marks)
b. Prove that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$.
(06 Marks)
c. Prove that $\beta(\mathrm{m}, \mathrm{n})=\frac{\Gamma_{\mathrm{m}} \Gamma_{\mathrm{n}}}{\Gamma(\mathrm{m}+\mathrm{n})}$.
(07 Marks)

7 a. Solve : $\frac{d y}{d x}=e^{x-y}+x^{2} e^{-y}$.
b. Solve $x^{2} y d x-\left(x^{3}+y^{3}\right) d y=0$.
(07 Marks)
c. Solve $\frac{d y}{d x}+y \cot x=\cos x$.
(06 Marks)
(07 Marks)

8 a. Solve : $\frac{d^{2} y}{d x^{2}}+\frac{4 d y}{d x}+4 y=0$.
b. Solve $\frac{d^{2} y}{d x^{2}}-\frac{6 d y}{d x}+9 y=3 e^{-4 x}$.
(05 Marks)
c. Solve : $y^{\prime \prime}+2 y^{\prime}+y=e^{-x}+\cos 2 x$.
d. Solve : $\frac{d^{2} y}{d x^{2}}-4 y=x \sin 2 x$.

